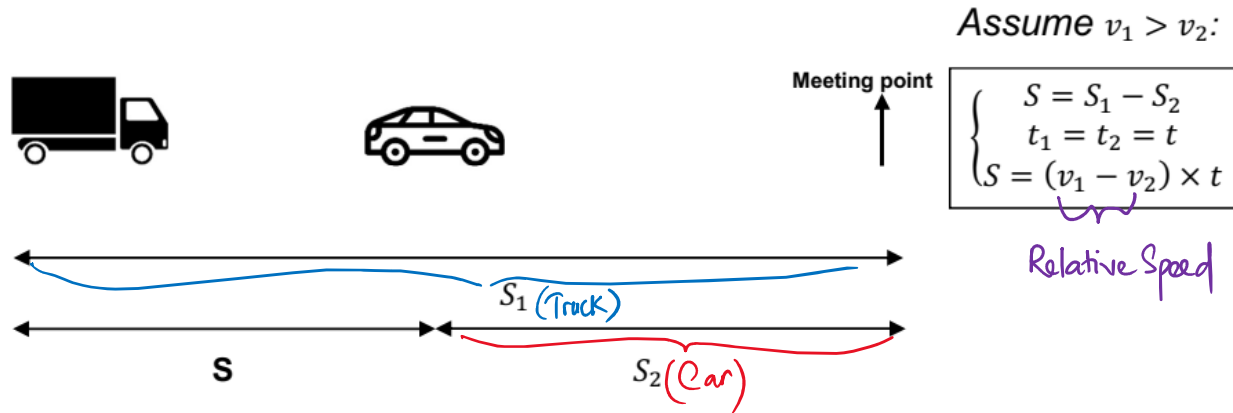


DISTANCE PATTERNS (PART 2 – OVERTAKING SCENERIO)

* Two subjects are far apart and move in the same direction on the same road. They will meet at a point after t units of time.

**Takeaways:**

1) When two trains are going in the same direction, then **their relative speed is the difference between the two speeds.**

2)

. When a train crosses a **stationary** man/ pole/ lamp post/ sign post- in all these cases, the object which the train crosses is stationary and **the distance travelled** is the length of the train.

. When it crosses a platform/ bridge- in these cases, the object which the train crosses is **stationary** and **the distance travelled** is the length of the train and the length of the object.

3) When two trains are moving in same direction, then their speed will be subtracted. The total distance is the sum of the length of both the trains.

4) When a train crosses a car/ bicycle/ a **mobile** man, **the relative speed** between the train and the object is taken depending upon the direction of the movement of the other object relative to the train **and the distance travelled** is the length of the train.

2. OVERTAKING SCENERIO

Ex 1: Mary passed a certain gas station on a highway while traveling west at a constant speed of 50 miles per hour. Then, 15 minutes later, Paul passed the same gas station while traveling west at a constant speed of 60 miles per hour. If both drivers maintained their speeds and both remained on the highway for at least 2 hours, how long after he passed the gas station did Paul catch up with Mary?

- A. 30 min
- B. 45 min
- C. 1 hr
- D. 1 hr 15 min
- E. 1 hr 30 min

Lưu ý: MP = khoảng cách giữa P và M = đoạn đường mà M đi trong 15'

$v_P = 60$ $v_M = 50$

(gas station) P _____ M _____ Overtaking point = C

$$15' \times v_M \\ = 15' \times 50 \text{ mi/h}$$

$t_P = ?$

Gọi t là thời gian mà P đi hết đoạn đường PC thì t cũng chính là thời gian M đi hết đoạn đường MC

$$MP = PC - MC \rightarrow 15' \times 50 = t \times v_P - t \times v_C = t \times (50 - 40) \rightarrow t = 15' \times 5 = 75' \rightarrow D$$

Similar questions:

. John travels westward past a certain bus stop at a constant speed of 40 miles per hour at 12 : 00 pm. Then, 10' later, Tom passed the same bus stop westward at a constant speed of 50 miles per hour. If both travelled at their constant rates, at what time after John passes the bus stop will Tom catch up with John?

Keys: 12 : 50 pm (lưu ý, thời gian mà John và Tom cùng xuất phát là lúc Tom passed the bus stop at 12:10 pm và thời gian cần để Tom đuổi kịp John tính được là 40')

. Sally left her home at 11am., traveling along Route 1 at 30 mph. At 1pm, her brother Tom left home and started after her on the same road at 45mph. At what time did Tom catch up to Sally?

Keys: 5 pm

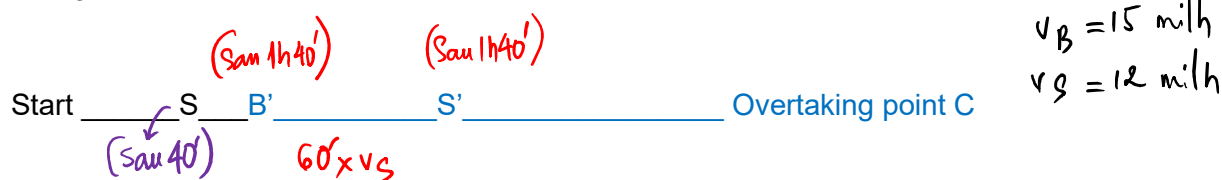
. Jacob was driving on a straight highway, from Alphaville to Betaville, at 60 mph. Rosalia was driving faster than Jacob, on the same highway, in the same direction. She started behind Jacob. She passed Jacob at noon. At 2:00 pm, Rosalia arrived at Betaville, and Jacob arrived

50 minutes later. Assume both drivers kept up constant speeds without stops. What was Rosalia's speed, in mph?

Keys: 85 mph

Ex 2: Ben and Sam set out together on a bicycle traveling at 15 and 12 miles per hour, respectively. After 40 minutes, Ben stops to fix a flat tire. If it takes Ben one hour to fix the flat tire and Sam continues to ride during this time, how many hours will it take Ben to catch up Sam and assuming he resumes his ride at 15 miles per hour?

- A. 3
- B. 3.33
- C. 3.5
- D. 4
- E. 4.5



Lưu ý, vì B chạy nhanh hơn S nên sau 40' S vẫn đang ở phía sau B dẫn tới khoảng cách giữa 2 người lúc xuất phát cùng thời điểm lần 2 không đúng bằng đoạn đường S đi trong 1h mà B dừng lại.

Gọi t là thời gian B cần đi để đuổi kịp Sam (đoạn $B'C$) cũng chính là thời gian Sam đi đoạn $S'C$.

$$t = ?$$

Ta cần xác định rõ, khoảng cách giữa B và S khi 2 người xuất phát cùng thời điểm lần thứ 2 = $B'S'$

$B'S' = SS'$ (thời gian S đi trong 1h mà B dừng) – SB' (chênh lệch khoảng cách giữa 2 người trong 40')

$$\text{Vậy: } B'S' = 1 \times 12 - \frac{40'}{60} \times (15 - 12) = t \times (15 - 12) \rightarrow t = 3.33 \rightarrow B$$

Similar question: Pam and Cathy begin running at the same time on a straight path. Pam runs at 10 miles per hour, and Cathy runs at 8 miles per hour. After 45 minutes, Pam stops to stretch. If it takes Pam 30 minutes to stretch and Cathy continues to run during this time, how many minutes will it take Pam to catch up to Cathy assuming Pam resumes running at 10 miles per hour?

Keys: 75 minutes

Ex 3: Richard began driving from home on a trip averaging 30 miles per hour. How many miles per hour must Carla drive on average to catch up to him in exactly 3 hours if she leaves 30 minutes after Richard?

A. 35

B. 55

C. 39

D. 40

E. 60

C _____ R _____ Overtaking point C

$$30' \times v_R$$

$$t = 30' \rightarrow v_C = ?$$

$$\frac{30'}{60} \times 30 = 3 \times (v_C - 30) \rightarrow v_C = 35 \rightarrow A$$

Ex 4: Andrew and Stephen drive on the highway in the same direction at respective rates of 72 and 80 kmh. If Stephen is 4 km behind Andrew, by how much does he have to increase his speed to catch up with Andrew in 20 minutes?

A. 1 kmh

B. 2 kmh

C. 3 kmh

D. 4 kmh

E. 5 kmh

$$v_S = 80 \quad v_A = 70$$

S _____ A _____ Overtaking point C

$$4 \text{ km}$$

$$t = 20'$$

Gọi x là lượng vận tốc cần tăng thêm của S $\rightarrow x = ?$

$$4 = \frac{20}{60} (v_S + x - 72) = \frac{1}{3} (80 + x - 72) \rightarrow x = 4 \rightarrow D$$

Ex 5: Car X is 40 miles west of Car Y. Both cars are traveling east, and Car X is going 50% faster than Car Y. If both cars travel at a constant rate and it takes Car X 2 hours and 40 minutes to catch up to Car Y, how fast is Car Y going?

- A. 20 miles per hour
- B. 25 miles per hour
- C. 30 miles per hour
- D. 35 miles per hour
- E. 40 miles per hour

$$v_X = 1.5v_Y$$

X _____ Y _____ Overtaking point C
 40 mi

$$t = 2h40' = 2 + \frac{40'}{60'} = 2\frac{2}{3} = \frac{8}{3}h$$

$$v_Y = ?$$

$$40 = \frac{8}{3} \times (v_X - v_Y) = \frac{8}{3} \times 0.5v_Y \rightarrow v_Y = 30 \rightarrow C$$

Similar question:

. Carol started from home on trip averaging 30 miles per hour. How fast must her mother drive to catch up her in 3 hours if she leaves 30 minutes after Carol?

Keys: 35 mi/h

. Car X and Y are traveling from A to B on the same route at constant speeds. Car X is initially behind Car Y, but Car Y's speed is 0.8 times Car X's speed. Car X passes Car Y at noon. At 1:45 pm, Car X reaches B, and at that moment, Car Y is still 35 miles away from B. What is the speed of Car X in miles per hour?

Keys: 100 mi/h

Overtaking point _____ Y' _____ X'
 12pm 35mi 1:45pm

$$t = 1\frac{3}{4}h = \frac{7}{4}h \quad / \quad v_Y = 0.8v_X$$

$$35 = \frac{7}{4} \times (v_X - v_Y)$$

. A car and truck are moving in the same direction on the same highway. The truck is moving at 50 miles an hour, and the car is traveling at a constant speed. At 3 pm, the car is 30 miles behind the truck and at 4:30 pm, the car overtakes and passes the truck. What is the speed of the car?

Keys: 70 mi/h

Ex 6: Ashok and Brian are both walking east along the same path; Ashok walks at a faster constant speed than does Brian. If Brian starts 30 miles east of Ashok and both begin walking at the same time, how many miles will Brian walk before Ashok catches up with him?

(1) Brian's walking speed is twice the difference between Ashok's walking speed and his own.

(2) If Ashok's walking speed were five times as great, it would be three times the sum of his and Brian's actual walking speeds.

$$v_A > v_B$$

A _____ B _____ Overtaking point C
 30 mi

$$BC = t \times v_B = ?$$

$$\text{Ta có: } AB = 30 = t(v_A - v_B)$$

$$1) v_B = 2(v_A - v_B) \quad v_A > v_B$$

$$\text{Thế vào đề: } \rightarrow 30 = t \times \frac{1}{2} v_B \Rightarrow t \times v_B = 30 \times 2$$

→ Sufficient

$$2) 5v_A = 3(v_A + v_B) \Rightarrow 2v_A = 3v_B$$

$$\text{Thế vào đề: } \Rightarrow 30 = t \times \left(\frac{2}{3}v_B - v_B\right) = t \times \frac{1}{3}v_B \rightarrow t \times v_B = 30 \times 3$$

→ Sufficient → D

Ex 7: Bob and Wendy planned to walk from their home to a restaurant for dinner together. However, Bob was delayed at work, and Wendy left for the restaurant before Bob did. If the restaurant is 3 miles from their home and Bob left for the restaurant a half-hour after Wendy did, how long did Wendy have to wait for Bob at the restaurant?

(1) Wendy walked at a constant pace of 4 miles per hour

(2) Bob walked at a constant pace of 1 mile per hour faster than Wendy.

 3 mi
 B _____ W _____ Restaurant
 30' < v_W

$$t_B - t_W = \left(\frac{3}{v_B} + 30'\right) - \frac{3}{v_W} = \boxed{\frac{3}{v_B} + \frac{1}{2} - \frac{3}{v_W} = ?}$$

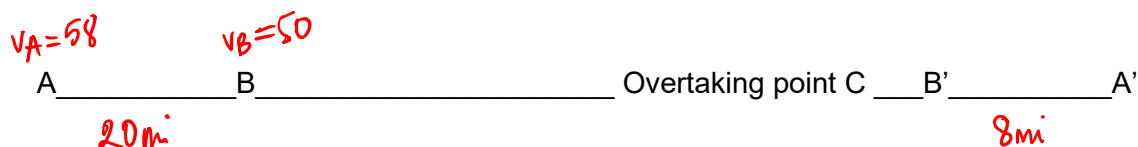
1) $v_W = 4 \rightarrow$ Vẫn chưa biết $v_B \rightarrow$ Insuff.

2) $v_B - v_W = 1 \rightarrow$ Thế vào ta vẫn còn ẩn $\rightarrow$ Insuff.

1) + 2) Ta giải được $v_B = 5$ và $v_W = 4 \rightarrow$ Tính được biểu thức được hỏi $\rightarrow$ Suff. $\rightarrow$ C

Ex 8: Car A is 20 miles behind car B, which is traveling in the same direction along the same route as Car A. Car A is traveling at a constant speed of 58 miles per hour and Car B is traveling at a constant speed of 50 miles per hour. How many hours will it take for Car A to overtake and drive 8 miles ahead of Car B?

- A. 1.5
- B. 2.0
- C. 2.5
- D. 3.0
- E. 3.5



$$t_{\text{overtake}} + t_{\text{ahead}} = ?$$

. **Bước 1:** Tìm $t_{\text{overtake}} = t_1$

Ta có: $AB = 20 = t_1(58 - 50) = 8t_1 \rightarrow t_1 = 2.5$ hours

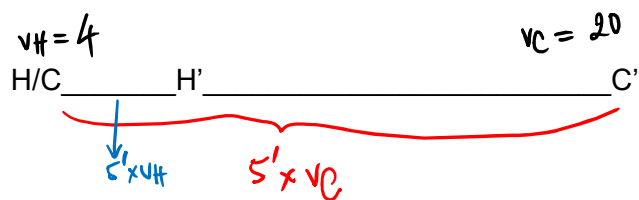
. **Bước 2:** Tìm $t_{\text{ahead}} = t_2$

Ta có: $B'A' = 8 = CA' - CB' = t_2(58 - 50) \rightarrow t_2 = 1$ hour

Vậy đáp án là 3.5 hours $\rightarrow$ E

Ex 9: A hiker walking at a constant rate of 4 miles per hour is passed by a cyclist traveling in the same direction along the same path at a constant rate of 20 miles per hour. The cyclist stops to wait for the hiker 5 minutes after passing her, while the hiker continues to walk at her constant rate. How many minutes must the cyclist wait until the hiker catches up?

- A. $6 \frac{2}{3}$
- B. 15
- C. 20
- D. 25
- E. $26 \frac{2}{3}$



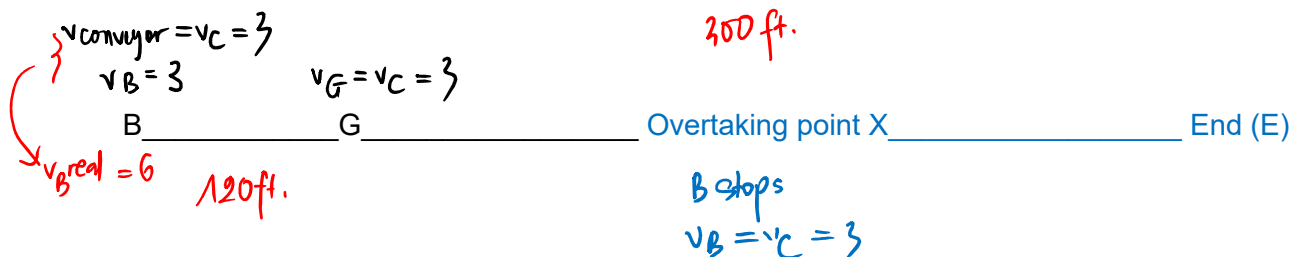
$$t_{H/C'} = ?$$

Trong 5' mà C đi rồi dừng thì H cũng đi đoạn HH' nên thời gian C cần chờ chỉ là cho H đi hết đoạn H'C' mà thôi.

Ta có: $H'C' = 5' \times 20 - 5' \times 4 = 80$ mà $H'C' = v_H \times t_{H/C'} = 4 \times t_{H/C'} = 80 \rightarrow t_{H/C'} = 20 \rightarrow C$

Ex 10: The 'moving walkway' is a 300-foot long walkway consisting of a conveyor belt that moves continuously at 3 feet per second. When Bill steps on the walkway, a group of people that are also on the walkway stands 120 feet in front of him. He walks toward the group at a rate of 3 feet per second. Once Bill reaches the group of people, he stops walking and stands with them until the walkway ends. What is Bill's average rate of movement for his trip along the moving walkway?

- A. 2 feet per second
- B. 2.5 feet per second
- C. 3 feet per second
- D. 4 feet per second
- E. 5 feet per second



$$v_B \text{ average} = \frac{300}{t_{\text{overtake}} + t_{XE}} = ?$$

Lưu ý $v_B \text{ real} = v_B + v_C = 3 + 3 = 6$ (trước khi B dừng)

. **Bước 1:** Tìm $t_{\text{overtake}} = \frac{120}{6-3} = 40$ seconds

. **Bước 2:** Tìm $t_{XE} = \frac{XE}{v_B} = \frac{300 - BX}{3} = \frac{300 - 40 \times 6}{3} = \frac{60}{3} = 20$ seconds

Như vậy $v_B \text{ average} = \frac{300}{40+20} = 5$ ft. per sec $\rightarrow E$

Ex 11: How many seconds will a 150 m long train running at a speed of 60 kmph take to cross a bridge of 300 m?

$$t = ?$$

$$60 \text{ kmh} = 60 \times \frac{1000}{3600} = 60 \times \frac{5}{18} = \frac{50}{3} \text{ m/s}$$

$$\text{Distance} = 150 + 300 = 450 \text{ m} = t \times v_{\text{train}} = \frac{50}{3}t \rightarrow t = 27\text{s}$$

Keys: 27 seconds

Takeaway:

When it crosses a platform/ bridge- in these cases, the object which the train crosses is **stationary** and the distance travelled is the length of the train and the length of the object.